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Boundary element solution of unsteady magnetohydrodynamic duct flow with differential quadrature time integration scheme

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SUMMARY

A numerical scheme which is a combination of the dual reciprocity boundary element method (DRBEM) and the differential quadrature method (DQM), is proposed for the solution of unsteady magnetohydrodynamic (MHD) flow problem in a rectangular duct with insulating walls. The coupled MHD equations in velocity and induced magnetic field are transformed first into the decoupled time-dependent convection–diffusion-type equations. These equations are solved by using DRBEM which treats the time and the space derivatives as nonhomogeneity and then by using DQM for the resulting system of initial value problems. The resulting linear system of equations is overdetermined due to the imposition of both boundary and initial conditions. Employing the least square method to this system the solution is obtained directly at any time level without the need of step-by-step computation with respect to time. Computations have been carried out for moderate values of Hartmann number ($M \leqslant 50$) at transient and the steady-state levels. As M increases boundary layers are formed for both the velocity and the induced magnetic field and the velocity becomes uniform at the centre of the duct. Also, the higher the value of M is the smaller the value of time for reaching steady-state solution. Copyright © 2005 John Wiley & Sons, Ltd.

KEY WORDS: dual reciprocity BEM; DQM; MHD flow equations

1. INTRODUCTION

The study of the flow of conducting fluids in the presence of transverse magnetic field has attracted attention owing to its applications in geology, power generation, flowmetry, thermonuclear reactor technology, etc. In general, the problems of magnetohydrodynamic (MHD) flow are extremely complex since they involve both the hydrodynamic flow equations and Maxwell equations of electrodynamics. The conducting fluid flow can induce electric current

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and interact with the magnetic field. This interaction in turn produces Lorentz force on the fluid and can greatly change the flow behaviour resulting in sharply varying induced magnetic field. Due to the coupling of hydrodynamic and magnetic fields, exact solutions have been found only for simple cases and geometry. Therefore, it is important to devise effective numerical methods for the approximate solution of MHD duct flow problems.

Many researchers investigated the MHD problem in 2-dimensional case using several numerical methods. Singh and Lal [1, 2] have obtained numerical solution of the MHD flows through pipes of triangular cross-section using a finite difference method (FDM). Then they presented the finite element method (FEM) solution of MHD flow in channels of arbitrary wall conductivity [3–5]. Singh and Lal's MHD duct problem solutions were only for Hartmann numbers less than 10. Tezer-Sezgin and Köksal [6] extended these studies to moderate Hartmann numbers by using FEM with linear and quadratic elements. Scandiuzzi and Schrefler [7] presented a FEM for solving 2-dimensional MHD flows and developed an equation relating magnetic field boundary conditions to the conductivity of the walls. Later, Demendy and Nagy [8] have used the analytical FEM to obtain their numerical solution of MHD flow in the range of Hartmann number <1000. The boundary element method application of the steady MHD flow in channels was considered by Singh and Agarwal [9] in terms of a singular integral equation. Carabineanu *et al.* [10], Tezer-Sezgin [11] and Liu and Zhu [12] have solved steady MHD duct flow problem for small and moderate values of Hartmann number.

Although steady flows have been studied extensively, a few papers have appeared on transient 2-dimensional MHD flows in channels. Among the papers mentioned above, Singh and Lal's [5] and Tezer-Sezgin and Köksal's [6] studies were on the FEM solution of time-dependent MHD flow equations. Seungsoo and Dulikravich [13] proposed a FDM scheme for 3-dimensional unsteady MHD flow together with temperature field. They have used explicit Runge–Kutta method for step-by-step computations in time. Sheu and Lin [14] presented a convection–diffusion–reaction model for solving unsteady MHD flow applying a FDM on non-staggered grids with a transport scheme in each ADI (predictor–corrector) spatial sweep. The stabilized FEM for the solution of 3-dimensional time-dependent MHD equations was given by Salah *et al.* [15]. The time derivative vectors were evaluated by a finite-difference-like expression involving two previous time steps. The solution algorithm in each of these unsteady MHD flow studies is based on explicit time-stepping schemes starting with the given initial conditions. It is known that for explicit schemes, the time increment must be taken very small to deal with the stability problems, and therefore they are computationally expensive. Implicit time-stepping schemes are usually unconditionally stable and do not need small time increments but the overall discretization with respect to spatial and time variables makes the numerical scheme very complicated. The aim of this paper is to use differential quadrature method (DQM) in time discretization and obtain the solution directly at the required time level. The combined application of DRBEM for the spatial partial derivatives and DQM for the time derivative in solving diffusion problems was presented by Tanaka and Chen [16]. The resulting Lyapunov matrix equation was solved by Bartels–Stewart algorithm to reduce the computing effort of solving such matrix equations.

In this paper, the coupled velocity and magnetic field equations are firstly transformed into decoupled time-dependent convection–diffusion-type equations with non-conducting walls. Then dual reciprocity boundary element method (DRBEM) is applied treating the time derivative and the convection terms as the nonhomogeneity in the equations which is approximated by using linear and quadratic radial basis functions. DRBEM application to time-dependent convection–diffusion equation results in a system of first-order initial value problems. This initial value problem is discretized by Bozkaya [17] using DQM which results in an overdetermined linear system of equations due to the insertion of both initial and boundary conditions. Then the least square method is made use of to obtain the final square linear system of equations containing unknown nodal values at both discretized space and time points. The ordering of the solution vector is such that, it contains all the nodal values in the spatial domain from initial to required time level. This way, the resulting linear system of equations can be solved by any direct or iterative solver without any special treatment like in Lyapunov matrix equations. Thus, the solution procedure can be used with large time increments to obtain the solution directly at the required time level without the need of step-by-step computations in time.

2. BASIC EQUATIONS

The governing equations of MHD flow are obtained from Maxwell equations of electromagnetism and the basic equations of fluid mechanics. The unsteady, laminar, fully developed flow of viscous, incompressible and electrically conducting fluid in a rectangular duct, subjected to a constant and uniform applied magnetic field B_0 can be put in the following non-dimensional form [18]:

$$\begin{aligned} \nabla^2 V + M\frac{\partial B}{\partial x} &= -1 + \frac{\partial V}{\partial t} \\ \nabla^2 B + M\frac{\partial V}{\partial x} &= \frac{\partial B}{\partial t} \end{aligned} \tag{1}$$

in $\Omega \times [0, \infty)$ with the boundary conditions and the initial condition

$$\begin{aligned} V(x,y,t) = 0 \quad B(x,y,t) = 0 \quad (x,y) \in \partial\Omega \\ V(x,y,0) = 0 \quad B(x,y,0) = 0 \quad (x,y) \in \Omega \end{aligned} \tag{2}$$

The boundaries of the duct are assumed to be insulating. $V(x,y,t)$, $B(x,y,t)$ are the velocity and the induced magnetic field, respectively, M is the Hartmann number. The applied magnetic field B_0 is parallel to x-axis. $V(x,y,t)$, $B(x,y,t)$ are in the z-direction which is the axis of the duct. The fluid is initially at rest and then starts to move down the duct by applying a constant pressure gradient. As $t \to \infty$ we get the steady-state solution. Due to the physical and geometrical conditions in which the motion takes place, most of the studies are concentrated not on the original unsteady but on the steady MHD equations which have exact solutions for simple geometry and wall conductivity. However, it is important to see the behaviour of the solution at transient levels as approaching to the steady state. Thus, the original unsteady MHD equations are solved for a rectangular duct with insulating walls in this paper.

Equations (1) may be decoupled by the change of variables

$$U_1 = V + B, \quad U_2 = V - B \tag{3}$$

as

$$
\begin{aligned}
\nabla^2 U_1 + M\frac{\partial U_1}{\partial x} &= -1 + \frac{\partial U_1}{\partial t} \\
& \qquad\qquad (x, y, t) \in \Omega \times [0, \infty) \\
\nabla^2 U_2 - M\frac{\partial U_2}{\partial x} &= -1 + \frac{\partial U_2}{\partial t}
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
U_1(x, y, t) = 0 \quad U_2(x, y, t) = 0 \quad (x, y) \in \partial\Omega \\
U_1(x, y, 0) = 0 \quad U_2(x, y, 0) = 0 \quad (x, y) \in \Omega
\end{aligned}
\tag{5}
$$

It is possible to go back to the original unknowns V and B through Equation (3).

Now, both equations of (4) are time-dependent convection–diffusion-type equations with the only difference being $+M$ replaced by $-M$ in the second equation. Thus, the equation to be considered

$$\nabla^2 u = b(x, y, u_x, u_t) \tag{6}$$

where

$$b(x, y, u_x, u_t) = -1 + \frac{\partial u}{\partial t} - M\frac{\partial u}{\partial x} \tag{7}$$

in the first equation of (4) and

$$b(x, y, u_x, u_t) = -1 + \frac{\partial u}{\partial t} + M\frac{\partial u}{\partial x} \tag{8}$$

in the second equation of (4) with zero boundary and initial conditions in both of them.

3. DRBEM APPLICATION

The direct BEM is difficult for the Poisson-type equation (6) due to the presence of the nonhomogeneous term b on the right-hand side of the equation. This involves the evaluation of a domain integral. Thus, the DRBEM procedure, suggested by Wrobel *et al.* [19], is made use of which expands the nonhomogeneity in terms of a set of basis functions. These basis functions are related to Laplacian of some auxiliary functions, and thus, the domain integral on the right-hand side is also transformed to a boundary integral. The boundary only nature of BEM when combined with the discretization in the time direction will result fewer number of linearized equations than obtained in the case of interior methods (FDM, FEM).

Equation (6) is weighted through the region Ω as

$$\int_\Omega \nabla^2 u u^* \, d\Omega = \int_\Omega b u^* \, d\Omega \tag{9}$$

where $u^* = (1/2\pi)\ln(1/r)$ is the fundamental solution of Laplace equation, r is the distance between the source and the fixed points.

The nonhomogeneity can be approximated by means of a set of radial basis (coordinate) functions $f_j(x,y)$ as

$$b(x,y,u_x,u_t) \approx \sum_{j=1}^{N+L} \alpha_j(t) f_j(x,y) \tag{10}$$

where α_j are unknown coefficients depending on time, N and L are the numbers of boundary and selected internal nodes, respectively. The approximating functions $f_j(x,y)$ are linked with the particular solutions $\hat{u}_j$ of the equation $\nabla^2 \hat{u}_j = f_j$.

Substituting f_j's into Equation (10) and then applying Green's second identity to both sides of Equation (9) we obtain the matrix-vector equation

$$Hu - Gq = (H\hat{U} - G\hat{Q})\alpha \tag{11}$$

The $(N+L)\times(N+L)$ matrices G and H contain the integrals of the fundamental solution and its normal derivative, respectively, over the boundary elements, u and q are $(N+L)\times 1$ vectors for the solution and its normal derivative at the nodes. Each of the vectors $\hat{u}_j$ and $\hat{q}_j = \partial\hat{u}_j/\partial n$ is considered to be one column of the matrices $\hat{U}$ and $\hat{Q}$, respectively. The vector α is obtained from Equation (10) as $\alpha = F^{-1}b$ with the $(N+L)\times 1$ vector b and the $(N+L)\times(N+L)$ matrix F contains the coordinates function column vectors f_j.

When α is substituted back in Equation (11) with a similar approximation for the unknown $u = F\beta$, the convection terms are also approximated by using F matrix as

$$Hu - Gq = (H\hat{U} - G\hat{Q})F^{-1}\left\{-1 + \frac{\partial u}{\partial t} \pm M\frac{\partial F}{\partial x}F^{-1}u\right\} \tag{12}$$

and after rearranging, the following system of ordinary differential equations is obtained:

$$C\dot{u} + Au - Gq = -C\{-1\} \tag{13}$$

where the matrices A and C are as follows:

$$A = H \pm MC\frac{\partial F}{\partial x}F^{-1} \tag{14}$$

$$C = -(H\hat{U} - G\hat{Q})F^{-1} \tag{15}$$

The sizes of the matrices $\hat{U}$, $\hat{Q}$, C, A are $(N+L)\times(N+L)$. Now, from Equation (13) the standard form of the first-order initial value problem is obtained

$$\dot{u} + Bu = Dq - \{-1\} \tag{16}$$

where $B = C^{-1}A$ and $D = C^{-1}G$ are again $(N+L)\times(N+L)$ matrices.

4. DIFFERENTIAL QUADRATURE METHOD (DQM) FOR THE INITIAL VALUE PROBLEM

Now Equation (16) is a set of the first-order ordinary differential equations in time. With the given initial condition (5) it is a system of initial value problem. Various time marching schemes such as Euler, Runge–Kutta, FDM based on Taylor series method can be used for obtaining solution at a required time level or at the steady state. Explicit methods must use very small time increments due to the stability problems and then step-by-step computations for reaching a certain time level which is computationally expensive. Implicit methods are usually unconditionally stable but the final system of equations has a very large size when it is combined with the space discretization. The system of initial value problem resulting from diffusion and diffusion–convection problems is discretized using DQM by Tanaka and Chen [16] and Bozkaya [17], respectively, after the application of DRBEM in spatial domain. In Tanaka and Chen's [16] application, the resulting system was a Lyapunov matrix equation which was solved by the special Bartels–Stewart algorithm to reduce the computing effort of solving such matrix equations. We follow here Bozkaya's [17] formulation of DQM in time discretization which results in a system of linear equations containing the unknown at both discretized space and discretized time values. The procedure is stable, convergent and gives the solution directly at the required time level with very small number of discretized time points.

Application of DQM [20] to the initial value problem (16) discretizes the solution u in the time direction

$$\sum_{j=1}^{K} w_{ij}^{(1)} U_j + BU_i = D\bar{q}_i - \{-1\}, \quad i = 1, 2, \ldots, K \tag{17}$$

where the vectors $w_{ij}^{(1)}$ are the weighting coefficients for the first-order derivative [21], K is the number of time discretization points, $U_i = u(t_i)$ and $\bar{q}_i$ are actually the u and q vectors, respectively,

$$u = \{u_1, u_2, \ldots, u_N, \ldots, u_{N+L}\}, \quad q = \{q_1, q_2, \ldots, q_N, 0, \ldots, 0\}$$

at the ith time level

$$U_i = \{u_{1i}, u_{2i}, \ldots, u_{Ni}, u_{(N+1)i}, \ldots, u_{(N+L)i}\}, \quad \bar{q}_i = \{q_{1i}, q_{2i}, \ldots, q_{Ni}, 0, \ldots, 0\}$$

in which $u_{ji} = u_j(t_i)$ and $q_{ji} = q_j(t_i)$.

One can notice that Equation (17) gives a system of linear equations for each time level t_i $(i = 1, 2, \ldots, K)$

$$w_{i1}^{(1)} U_1 + w_{i2}^{(1)} U_2 + \cdots + w_{iK}^{(1)} U_K + BU_i = D\bar{q}_i - \{-1\} \tag{18}$$

where each U_i and $\bar{q}_i$ are of sizes $(N + L) \times 1$. When system (18) is written for $i = 1, 2, \ldots, K$, we finally obtain the system of linear equations for the solution of the convection–diffusion problem in the entire time domain

$$\tilde{A}\tilde{U} = \tilde{D}\tilde{q} - \{-1\} \tag{19}$$

where

$$\tilde{A} = W + \tilde{B} \tag{20}$$

The matrices $W, \tilde{B}$ and $\tilde{D}$ are expressed as

$$W = \begin{bmatrix} w_{11} & w_{12} & \dots & w_{1K} \\ w_{21} & w_{22} & \dots & w_{2K} \\ \vdots & & & \\ w_{K1} & w_{K2} & \dots & w_{KK} \end{bmatrix} \tag{21}$$

with $(N+L)\times(N+L)$ submatrices w_{ij} defined as $w_{ij} = w_{ij}^{(1)} I$, and

$$\tilde{B} = \begin{bmatrix} B & & & \\ & B & & \\ & & \ddots & \\ & & & B \end{bmatrix}, \quad \tilde{D} = \begin{bmatrix} D & & & \\ & D & & \\ & & \ddots & \\ & & & D \end{bmatrix} \tag{22}$$

The sizes of the matrices $\tilde{A}$, $\tilde{B}$, W and $\tilde{D}$ are $(N+L)K \times (N+L)K$ and the identity matrix I is of size $(N+L)\times(N+L)$.

The $(N+L)K \times 1$ vectors $\tilde{U}$ and $\tilde{q}$ are defined as

$$\tilde{U} = \{u_{11}, u_{21}, \dots, u_{(N+L)1}; u_{12}, u_{22}, \dots, u_{(N+L)2}; \dots; u_{1K}, u_{2K}, \dots, u_{(N+L)K}\} \tag{23}$$

$$\tilde{q} = \{q_{11}, q_{21}, \dots, q_{N1}, 0, \dots, 0; q_{12}, \dots, q_{N2}, 0, \dots, 0; \dots; q_{1K}, \dots, q_{NK}, 0, \dots, 0\} \tag{24}$$

In the linear system (19) boundary conditions (some of $\tilde{U}$ and some of $\tilde{q}$ nodal specified values) are inserted by interchanging the negative of corresponding columns and reordering the solution vector in terms of unknown $\tilde{U}$ and $\tilde{q}$ nodal values. When the initial condition is also inserted at the interior plus boundary nodes for the initial time level, system (19) finally becomes a rectangular system since known initial $\tilde{U}$ values are passed to the right-hand side leaving less number of unknowns than the number of equations.

The resulting reordered form of system (19) is given as

$$\tilde{\tilde{A}}\tilde{\tilde{U}} = \tilde{\tilde{D}}\tilde{\tilde{q}} - \{-1\} \tag{25}$$

where the sizes of $\tilde{\tilde{A}}, \tilde{\tilde{U}}$ are $(N+L)K \times ((N+L)K - L)$ and $((N+L)K - L) \times 1$, respectively. The sizes of $\tilde{\tilde{D}}$, $\tilde{\tilde{q}}$ are the same as the sizes of $\tilde{D}$ and $\tilde{q}$. Now $\tilde{\tilde{U}}$ contains all the unknown values of $\tilde{U}$ and $\tilde{q}$ but $\tilde{\tilde{q}}$ contains boundary plus initial information.

System (25) gives the solution of our time-dependent convection–diffusion problem at all the required time levels directly without the need of step-by-step computation in time and without the need of using small time increment. But since the system is overdetermined least square method must be employed to obtain the solution.

Application of the least square method to the overdetermined system (25) gives the normal equations

$$\tilde{\tilde{A}}^{\mathrm{T}}\tilde{\tilde{A}}\tilde{\tilde{U}} = \tilde{\tilde{A}}^{\mathrm{T}}\tilde{\tilde{D}}\tilde{\tilde{q}} - \tilde{\tilde{A}}^{\mathrm{T}}\{-1\} \tag{26}$$

for the unknown $\tilde{\tilde{U}}$ which is the solution of our problem (1) for the entire domain $\Omega \times (0, T)$ in which T denotes the steady-state level.

5. NUMERICAL RESULTS AND DISCUSSION

The time-dependent MHD equations defining viscous, laminar flow of an incompressible electrically conducting fluid in a square duct $|x| \leqslant 1$, $|y| \leqslant 1$ for the time domain $(0, T)$ are solved. In the DRBEM space discretization for the square domain, we use constant boundary elements

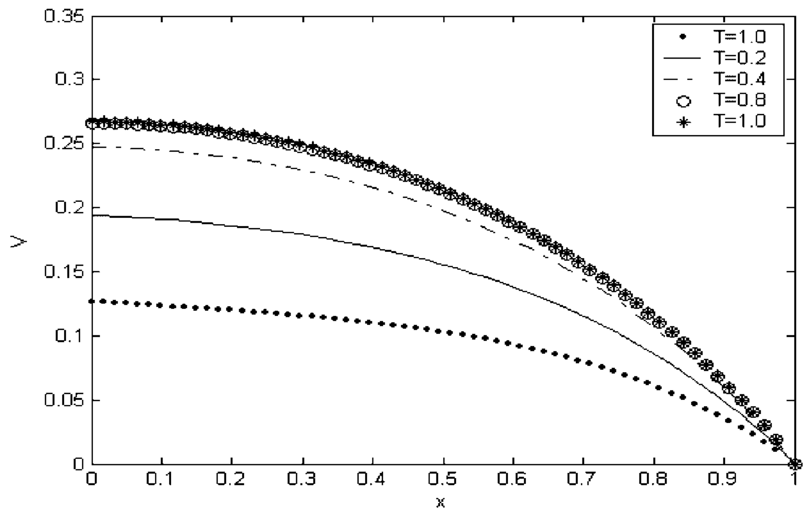

Figure 1. Velocity for $M = 2$, $N = 40$, $L = 60$.

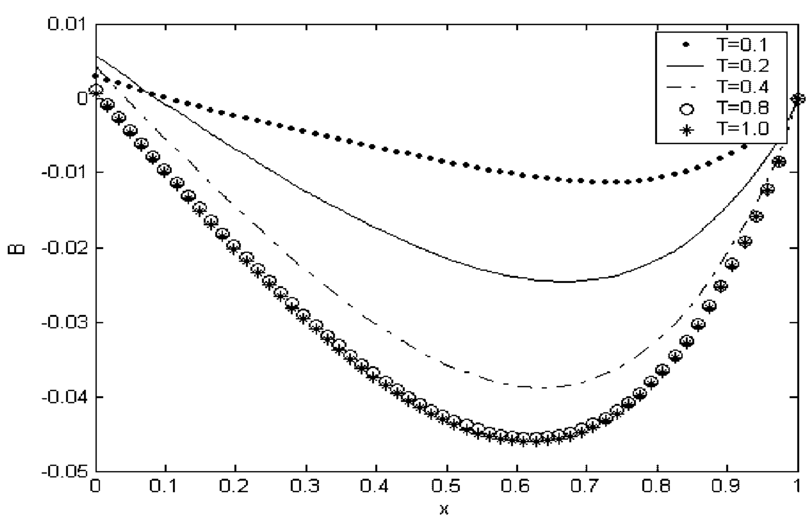

Figure 2. Magnetic field for $M = 2$, $N = 40$, $L = 60$.

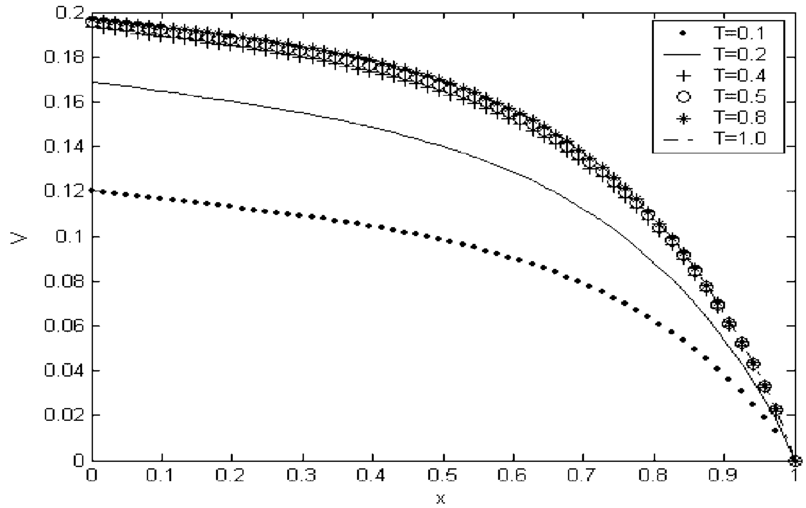

Figure 3. Velocity for $M=5$, $N=40$, $L=60$.

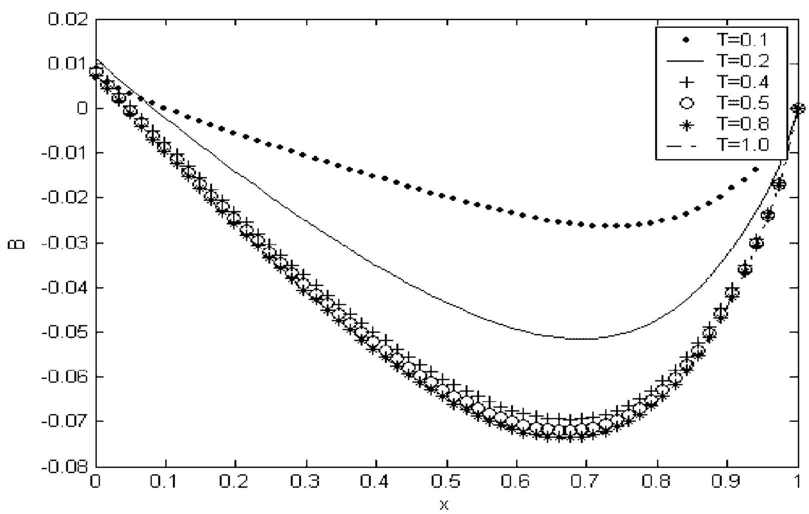

Figure 4. Magnetic field for $M=5$, $N=40$, $L=60$.

with the number of elements ranging from 40 to 320 and some equally spaced interior points for representing the solution in terms of graphics. The distribution of the interior points is arbitrary which is one of the advantage of the DRBEM. We may place more points close to the walls where the most of the action takes place in MHD flow. But the number of interior points (L) affects the accuracy of the solution since the size of the final system to be solved is $((N+L)K-L)\times((N+L)K-L)$, N and K are being the number of boundary and time discretized points. For the time domain $(0,T)$ in DQM Gauss–Chebychev–Lobatto

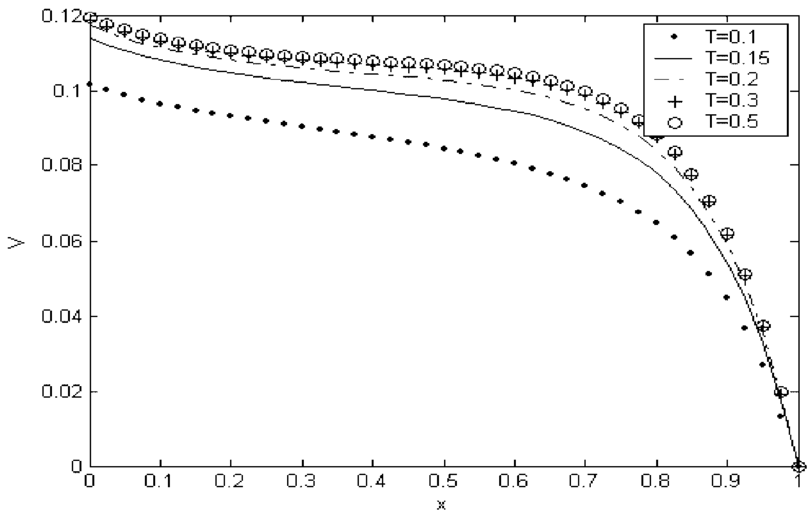

Figure 5. Velocity for $M=10$, $N=40$, $L=60$.

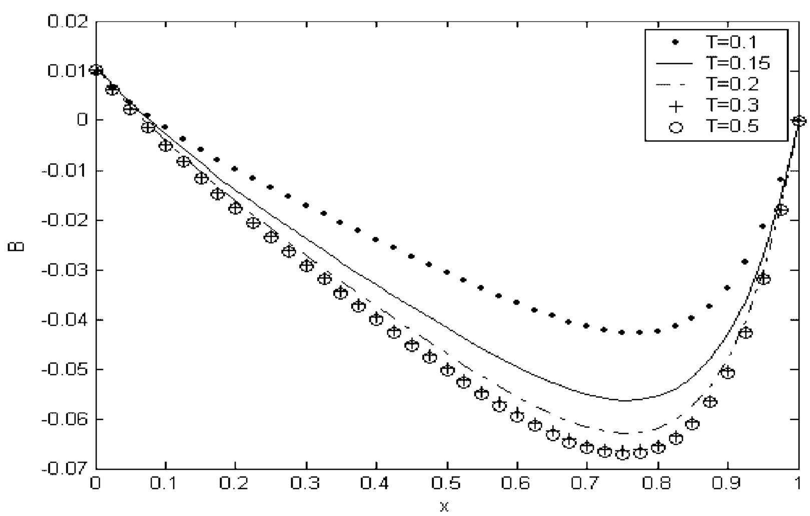

Figure 6. Magnetic field for $M=10$, $N=40$, $L=60$.

(G-C-L) points are used in the discretization. These points are non-uniform and clustered near the boundary which enable us to obtain converged and stable solution for our diffusion–convection-type equation [20]. Linear and quadratic radial basis functions are used in approximating the right-hand side function $b(x, y, u_x, u_t)$. Since we solve the transient MHD equations, we are able to obtain the solution at any required time level. As $t \to \infty$, we get the steady-state solution which can be compared with the available exact solution of the steady equations to check the accuracy of the results. Our steady-state results for the velocity and the induced

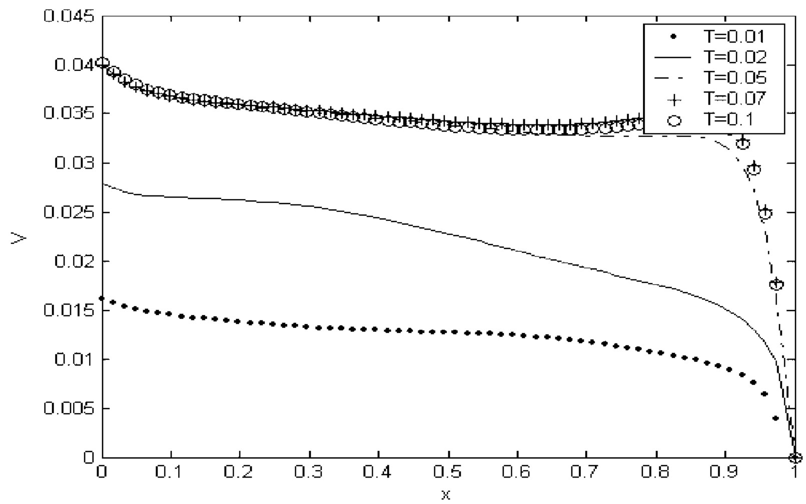

Figure 7. Velocity for $M = 30$, $N = 40$, $L = 60$.

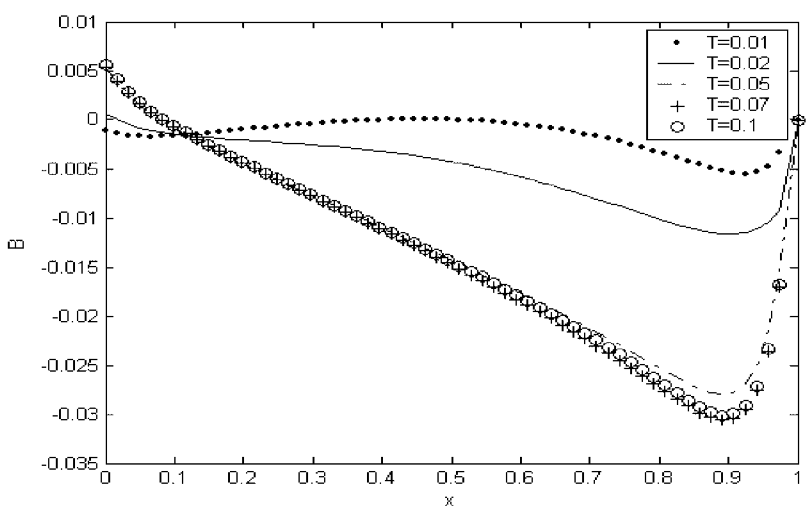

Figure 8. Magnetic field for $M = 30$, $N = 40$, $L = 60$.

magnetic field for small and moderate values of Hartmann number ($2 \leqslant M \leqslant 30$) agree with the Shercliff's [22] exact solution to roughly three significant digits.

Figures 1, 3, 5 and 7 exhibit the behaviour of velocity along the x-axis ($y = 0, 0 \leqslant x \leqslant 1$), respectively, for $M = 2, 5, 10$ and 30 at several time levels. Similarly, Figures 2, 4, 6 and 8 are induced magnetic field curves for the same Hartmann numbers and time levels. One can notice that steady-state values for the velocity and the induced magnetic field have been reached at a faster rate when Hartmann number is increased. The solution (V, B) remains in these steady-state values which are $T = 0.8, 0.4, 0.2$ and 0.05 for $M = 2, 5, 10$ and 30, respectively.

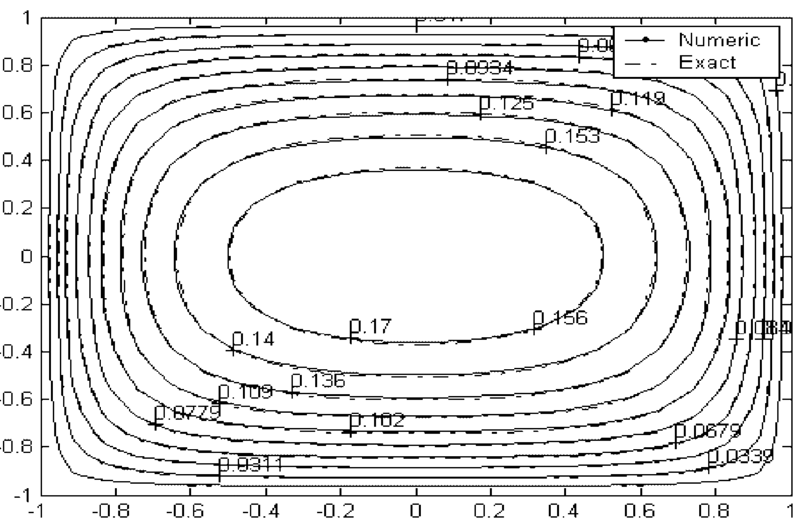

Figure 9. Velocity for $M = 5$, $N = 92$.

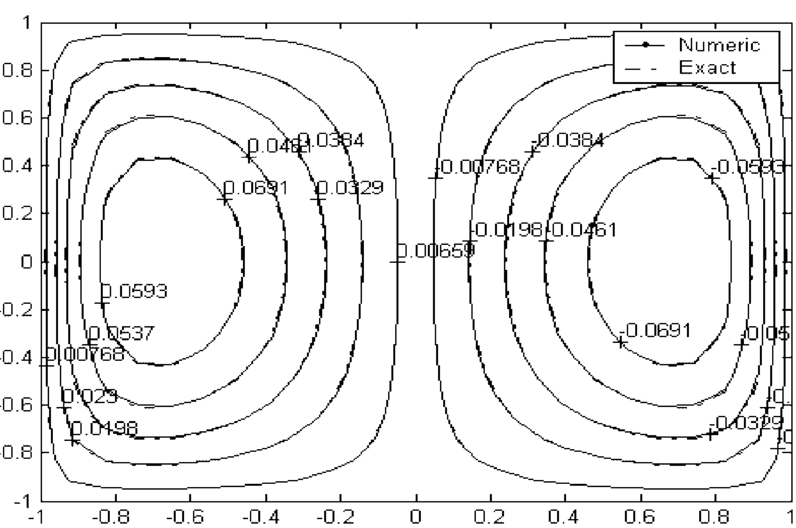

Figure 10. Magnetic field for $M = 5$, $N = 92$.

In Figures 9–10, 11–12 and 13–14 the equal-velocity–equal-induced magnetic field lines (contours) are presented at steady-state, respectively, for Hartmann numbers of $M = 5, 10$ and 20. It is noticed from Figures 9, 11 and 13 that, as the Hartmann number increases the velocity shows a flattening tendency (contour values are decreased). It is also observed that the boundary layer formation starts near the walls for both the velocity and the induced magnetic field for increasing Hartmann number. This is the well-known behaviour of MHD duct flow. For the larger value of M the thickness of the boundary layer is smaller. One

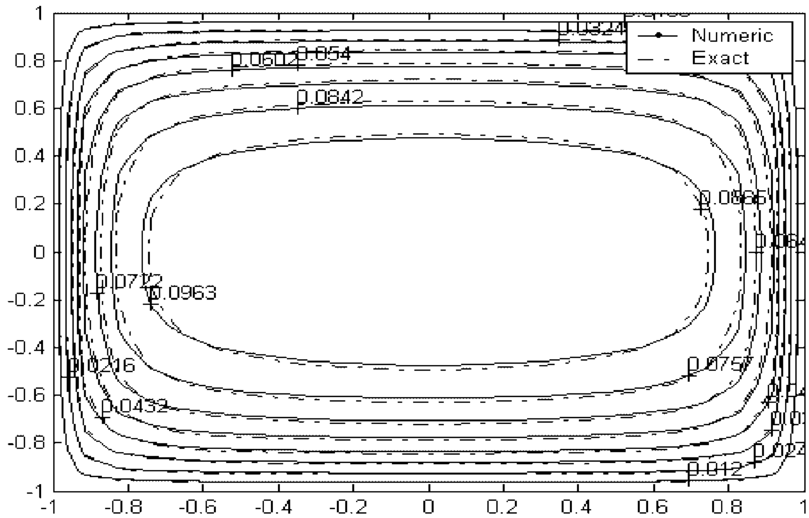

Figure 11. Velocity for $M = 10$, $N = 92$.

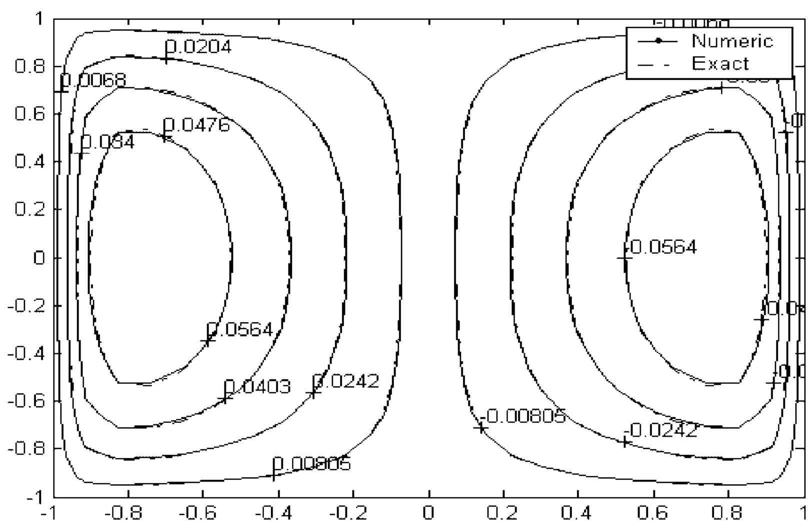

Figure 12. Magnetic field for $M = 10$, $N = 92$.

can also notice from Figures 9, 11 and 13 that, velocity becomes uniform at the centre of the duct when M is increased and it always has its maximum through the centre. When the Hartmann number is increased, the number of boundary elements (N) must be increased in our computations to get good accuracy. For small values of Hartmann number ($M \leqslant 10$) it is sufficient to take N around 60 but for $M = 20$ and $M = 30$ we need to take N as 108 and 240, respectively. In obtaining numerical solution for $M = 50$ we need resolution with an increase of boundary elements number N. At the same time, since we need efficient number

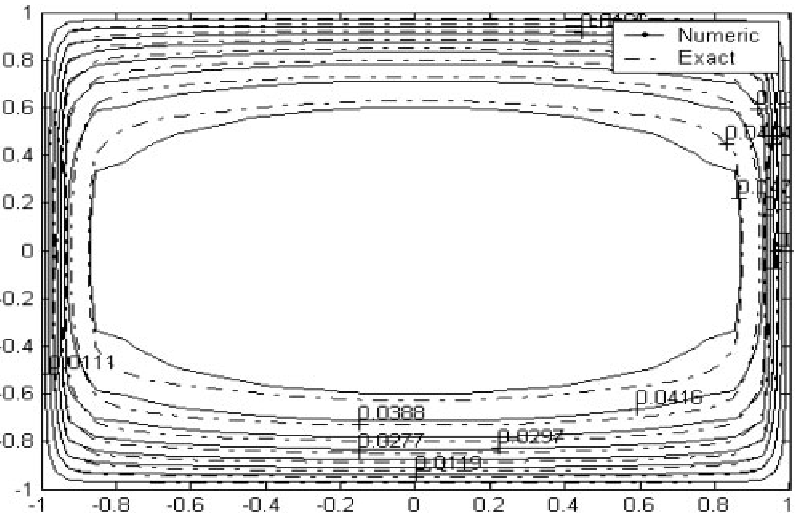

Figure 13. Velocity for $M = 20$, $N = 108$.

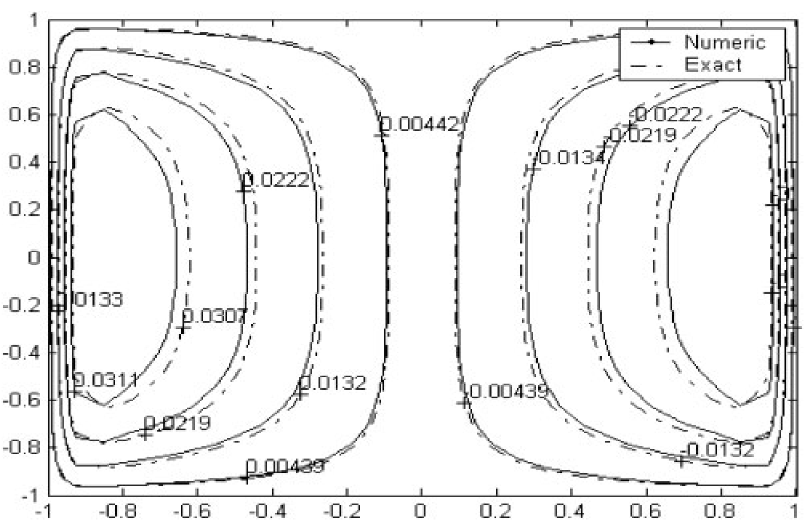

Figure 14. Magnetic field for $M = 20$, $N = 108$.

of interior points (L) for representing the solution in terms of graphics, we control the total size $((N+L)K-L)\times((N+L)K-L)$ of the system with the values of N, L and K not to have difficulties in solving large systems. For this reason we could increase Hartmann number up to 50 which needs 320 boundary elements. Further than this value of Hartmann number, the total size of the system becomes very large due to the large values of especially N, and K and L. Figures 15–16 and 17–18 are the velocity-induced magnetic field curves for $M = 30$

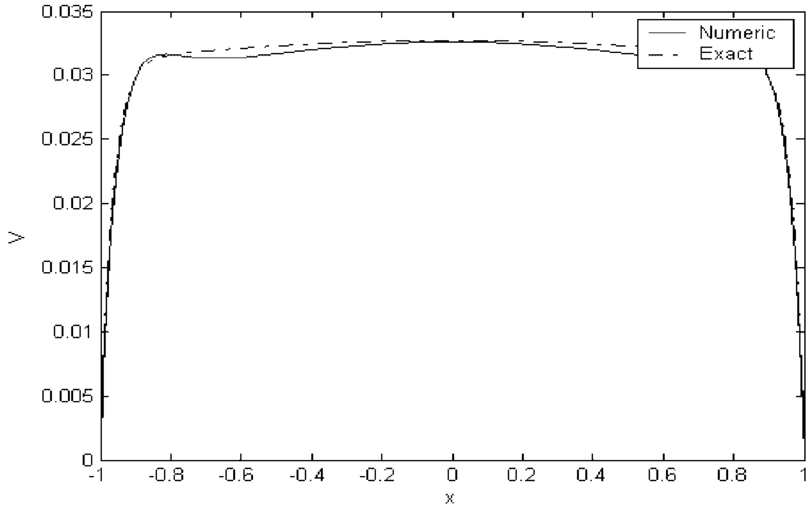

Figure 15. Velocity for $M = 30$, $N = 240$, $Y = 0.5$.

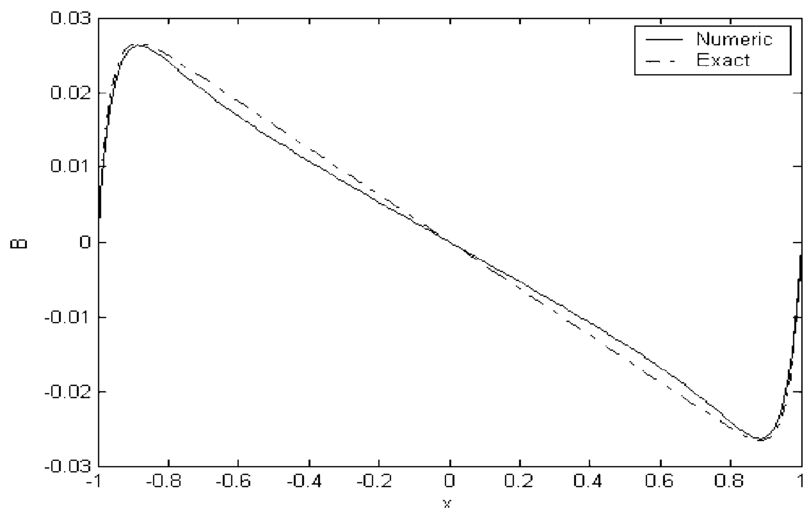

Figure 16. Magnetic field for $M = 30$, $N = 240$, $Y = 0.5$.

at $y = 0.5$ $(-1 \leqslant x \leqslant 1)$ and for $M = 50$ at $y = 0$ $(-1 \leqslant x \leqslant 1)$, respectively. As we increase Hartmann number M to values 50, discrepancies are examined especially for velocity. This may be due to the fact that we need to solve a larger sized matrix system resulting from the use of more boundary and interior points. Accumulation of roundoff errors drops accuracy to 10^{-2} especially close to the corners. In Figure 17 we notice that the numerical velocity values for $M = 50$ differ from the exact velocity values near the walls because of this accuracy drop. Increasing the mesh resolution seems to result in very slight improvements in the accuracy

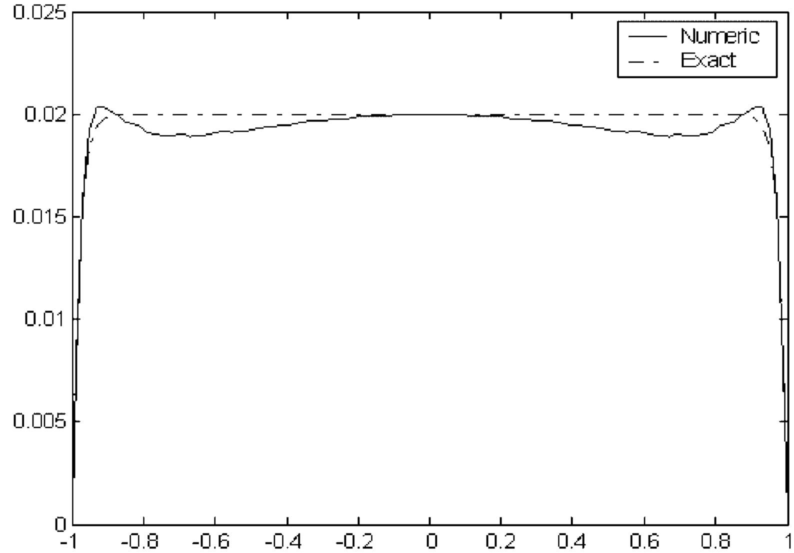

Figure 17. Velocity for $M = 50$, $N = 320$, $Y = 0$.

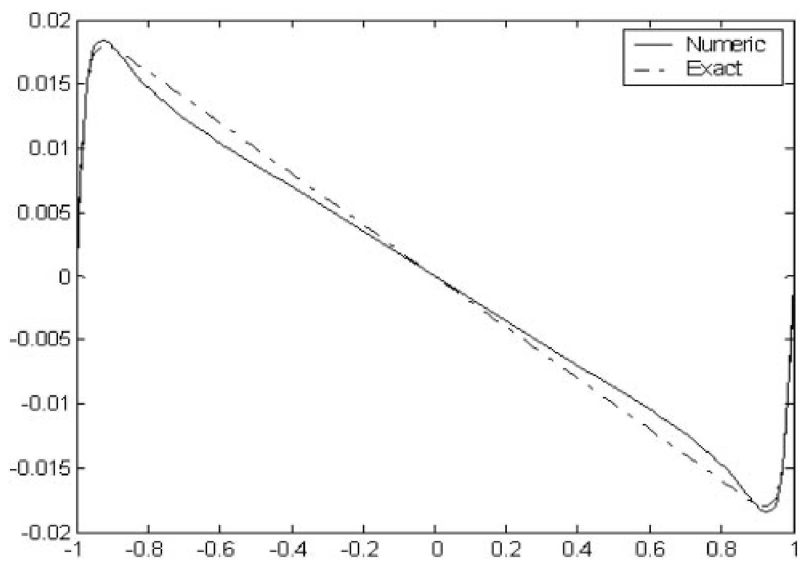

Figure 18. Magnetic field for $M = 50$, $N = 320$, $Y = 0$.

of the solution for $M = 50$. Thus, for Hartmann number values $M \geqslant 50$ the continuation with a mesh resolution is not practical due to the computational cost in the proposed numerical solution.

Time discretization points in the DQM application for the initial value problem (16) are taken as Gauss–Chebychev–Lobatto points which are mostly placed near the boundaries. Since the DQM is stable and convergent for these non-uniform points we don't need to take too many points. In the computations the number of G-C-L points was taken at most 5. The other explicit methods need very small time increments for the step-by-step computation in time (e.g. $\Delta t = 10^{-3}$ in Reference [14]). By taking T as the required time level and using G-C-L points in the interval $(0, T)$ the solution is obtained directly at $t = T$ from the solution of system (26).

6. CONCLUSION

The coupling of DRBEM and DQM for solving unsteady MHD duct flow problem gives a linear system of equations for the unknown nodal values at both discretized space and discretized time points. This way, the solution can be obtained at any required time level including the steady state, by using very small number of time discretization points and, without the need of a time-stepping scheme. The solution procedure outlined in this paper is applicable for MHD duct flow with nonconducting walls for which the equation can be decoupled and for the moderate values of Hartmann number ($M \leqslant 50$). Then the equations are treated as time-dependent diffusion–convection-type equations in the DRBEM. For conducting or partly conducting partly insulating walls, the equations must be solved by using BEM in coupled form for which the corresponding fundamental solution need to be derived.

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